A Computational Security Analysis of Signal's PQXDH Handshake

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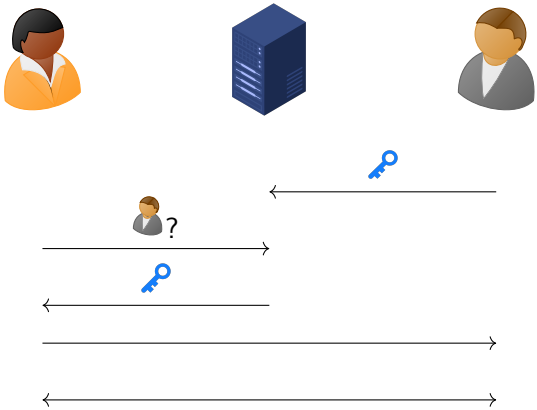
Asynchronous Authenticated Key Exchange



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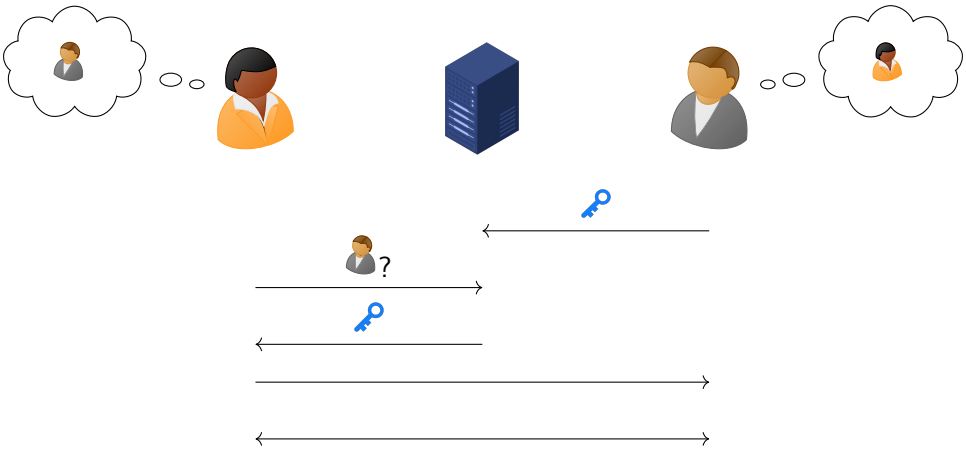

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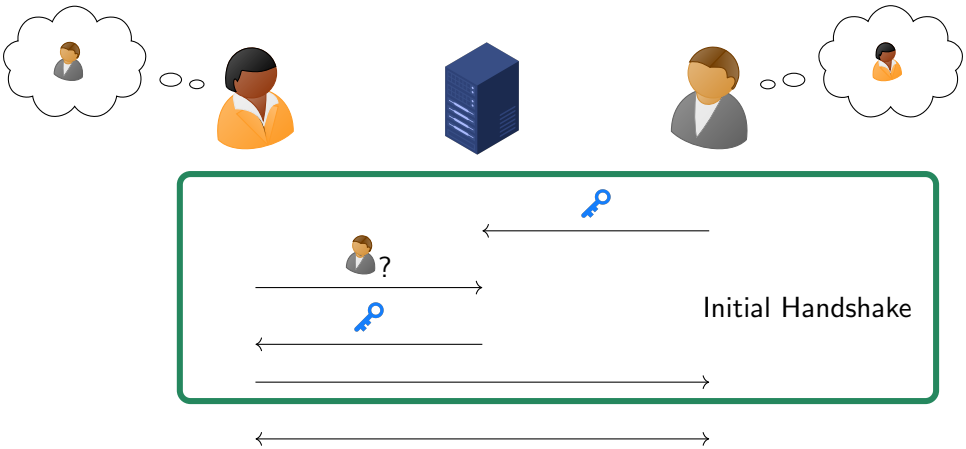
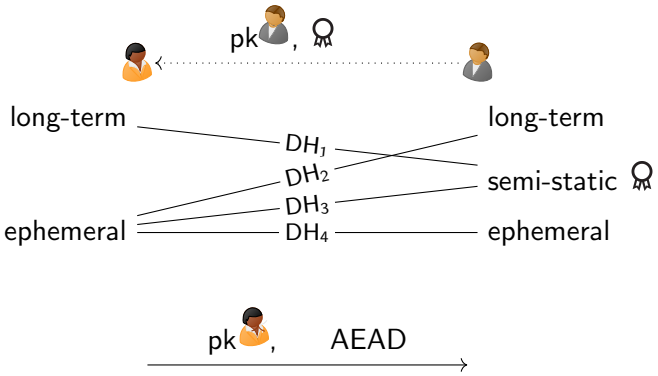


Signal's Initial Handshake(s): X3DH and PQXDH



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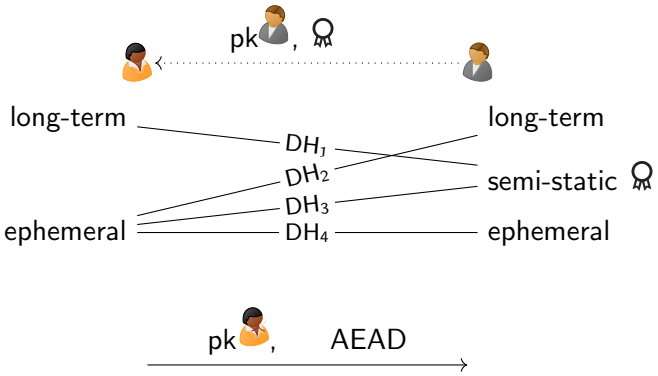

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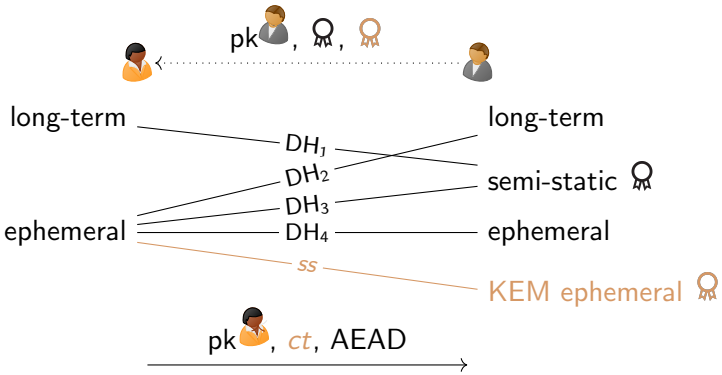

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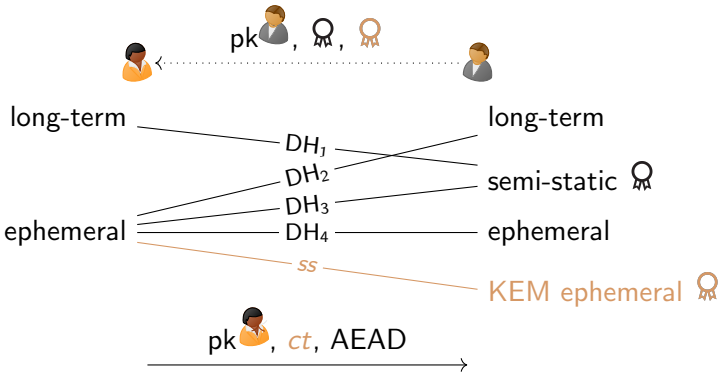

- session key: $\mathrm{KDF}(\mathrm{DH}_1 \| \ldots \| \mathrm{DH}_4 \| ss)$
- reduced session: Bob without ephemeral keys, semi-static KEM

Analyzing Signal's Initial Handshake(s): X3DH and PQXDH

- reductionist analysis of X3DH [CCD+17] with a [BR94] style key exchange model
- tool-based analysis of PQXDH with ProVerif and CryptoVerif [BJK23, BJKS23]
  - (re-)discovered (potential) KEM re-encapsulation attack [CDM23]
  - corruption of long-term keys only

[BJK23, BJKS23] Barghavan, Jacomme, Kiefer, Schmidt, 2023
[BR94] Bellare, Rogaway, CRYPTO 1993
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  - corruption of long-term keys only
- our work: [FG24]
  - follows [CCD+17, BFG+22] but explicitly models signatures (albeit with distinct signing keys)
  - identifies precise requirements of the KEM
  - models maximum-exposure w. clean predicates: $\underbrace{\underbrace{\underbrace{\mathsf{clean}_{\mathsf{LTSS}}, \mathsf{clean}_{\mathsf{ELT}}, \mathsf{clean}_{\mathsf{ESS}}}_{\text{reduced}}, \mathsf{clean}_{\mathsf{EE}}}_{\text{X3DH full}}, \mathsf{clean}_{\mathsf{sigE}}}_{\text{PQXDH full}}$

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Concrete Bound for PQXDH

$$\mathsf{Adv}^{\mathsf{KI}}_{\mathsf{PQXDH}}(\mathcal{A}) \leq \frac{(n_p + n_p \cdot n_{ss} + n_s)^2}{q} + \gamma_{\mathsf{coll}}(n_p \cdot n_{ss} + n_s) + n_s \cdot \delta_{\mathsf{corr}} + \epsilon_{LEAK^+}$$

$$+ \left(\begin{array}{ll} \quad (n_p \cdot (\epsilon_{\mathsf{SIG}} + n_p \cdot n_{ss} \cdot \epsilon_{\mathsf{GDH}})) & // \, \mathsf{clean}_{\mathsf{LTSS}} \\ + (n_s \cdot n_p \cdot \epsilon_{\mathsf{GDH}}) & // \, \mathsf{clean}_{\mathsf{ELT}} \\ + (n_p \cdot (\epsilon_{\mathsf{SIG}} + n_{ss} \cdot n_s \cdot \epsilon_{\mathsf{GDH}})) & // \, \mathsf{clean}_{\mathsf{ESS}} \wedge \mathsf{type} = \mathsf{full} \\ + (n_p \cdot (\epsilon_{\mathsf{SIG}} + n_{ss} \cdot n_s \cdot \min(\epsilon_{\mathsf{GDH}}, \epsilon_{\mathsf{CCA}}))) & // \, \mathsf{clean}_{\mathsf{ESS}} \wedge \mathsf{type} = \mathsf{reduced} \\ + (n_s^2 \cdot \min(\epsilon_{\mathsf{GDH}}, \epsilon_{\mathsf{CCA}})) & // \, \mathsf{clean}_{\mathsf{EE}} \wedge \mathsf{clean}_{\mathsf{peerE}} \\ + (n_p \cdot (\epsilon_{\mathsf{SIG}} + n_s^2 \cdot q_{\mathrm{RO}} \cdot \epsilon_{\mathsf{CCA}})) & // \, \mathsf{clean}_{\mathsf{EE}} \wedge \mathsf{clean}_{\mathsf{sigE}} \end{array} \right)$$

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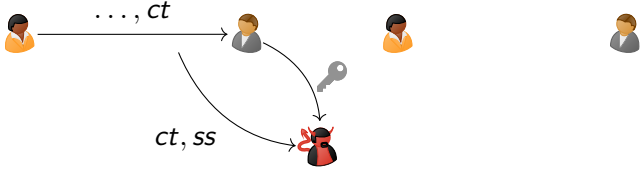

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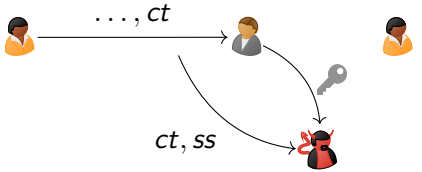

$\underline{KEM.encaps(\mathrm{pk}) :}$
$ss \leftarrow \{0,1\}^{256}$
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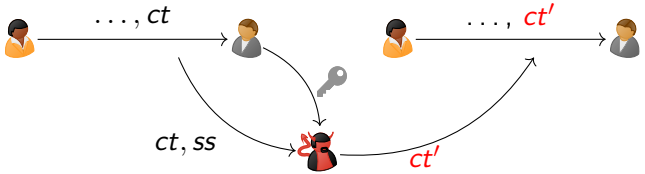

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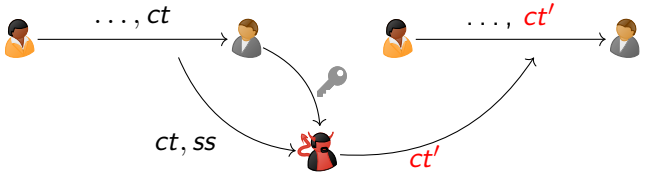

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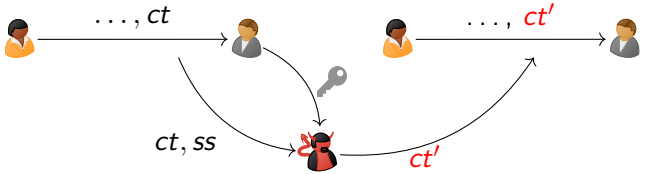

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- $\Rightarrow$ two sessions with same session key: adversary can reveal one and test the other
- [BJK23, BJKS23] models the KEM public key into the Associated Data of the AEAD
- which KEM property needed?
- proposed protocol fix: session context (KEM public key, ciphertext) in key derivation

KEM Binding Notion $LEAK^{+}$-BIND-SS-$\{CT, PK\}$ (extending [CDM23])

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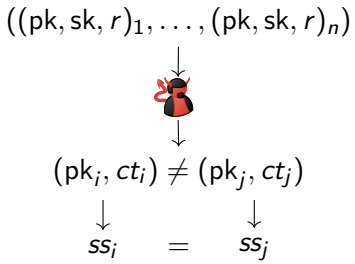

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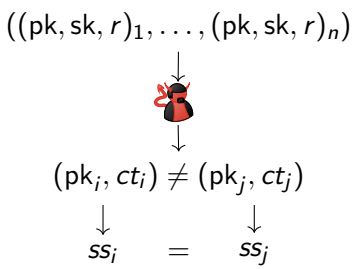

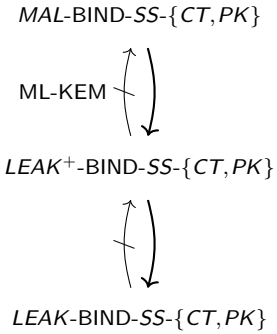

Concrete Hybrid Bound for PQXDH

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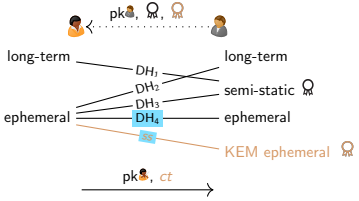

Concrete Bound for PQXDH Against Active-Later-Quantum Adversaries

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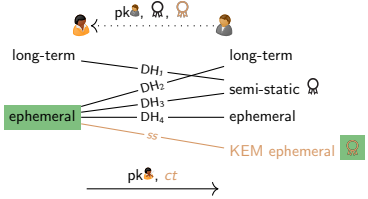
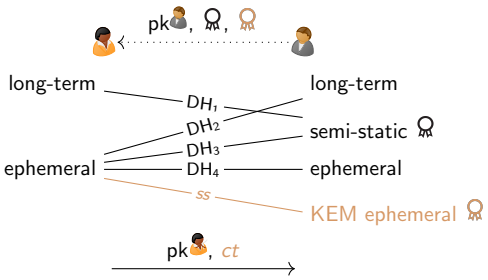
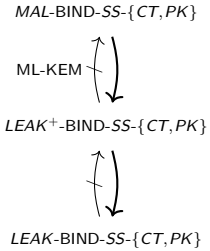


PQXDH Provides Hybrid Security





or include the session context in key derivation

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combine tool-based and reductionist analyses
⇒ detect attacks and identify requirements

`https://eprint.iacr.org/2024/702`

rune.fiedler@cryptoplexity.de

References I

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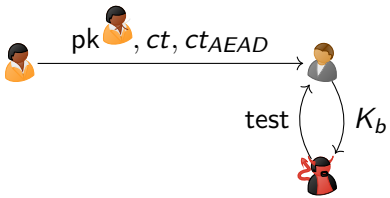
Picture References

- server icon by Alexiuz AS
- public key icon by Yannick Lung
- secret key icon by Yannick Lung
- signature icon by PINPOINT.WORLD

What if we model the AEAD ciphertext?



if $AEAD.Dec(K_b, ct_{AEAD}) \neq \perp$: **return** 0
else return 1

- $\Rightarrow$ adversary trivially wins (except with negligible probability)