

# **Computing on your data with MPC**

Christopher Patton CAW 2024



### **The tech industry needs data to operate**





### **The tech industry collects more data than it uses**





# **Data minimization**

Collect what you use and nothing more.

"Which users visited example.com  $m_1,\ldots,m_N$ measurements on Thursday" "How many users visited  $f(m_1,\ldots,m_N)$ aggregate example.com on Thursday"



# **The PPM working group at IETF**

- **IETF: "Internet Engineering Task Force"** 
	- Specifies many of the protocols that undergird the Internet (DNS, TLS, HTTP, …)
- PPM: "Privacy Preserving Measurement" working group
	- Lower the cost of data minimization
		- turn fancy crypto (MPC) into boring crypto
		- compute, bandwidth, dollars spent
	- Drive innovation by providing a deployment path for new research





# **The PPM working group at IETF**

- **2017:** Corrigan-Gibbs and Boneh propose [Prio](https://crypto.stanford.edu/prio/)
- **2018:** [Mozilla experiments with Prio](https://blog.mozilla.org/security/2019/06/06/next-steps-in-privacy-preserving-telemetry-with-prio/) for origin telemetry
- **2020**: [Google, Apple, and ISRG deploy Prio](https://covid19-static.cdn-apple.com/applications/covid19/current/static/contact-tracing/pdf/ENPA_White_Paper.pdf) for COVID-19 exposure notification apps
- **2021:** Working group formed
- **2022:** Working group adopts its first draft
- **2023:** [First deployments](https://blog.mozilla.org/en/products/firefox/partnership-ohttp-prio/) of DAP/Prio3 (candidate standard for Prio)
- **Goal for 2024: Finish the base drafts**





- **● VDAF: building a box around MPC**
- Building VDAFs
- Beyond VDAFs
- MPC hot takes  $\bigcirc$













First aggregator sums up its input shares to get its **aggregate share**





Second aggregator sums up its input shares to get its **aggregate share**





e collector sums up gregate shares to get *aggregate result*



- Prio: represent **aggregation function** as a linear function of (some encoding of) the measurements
- Not sufficient: **need interaction!**





### **Need for interactivity: input validation**

Secret sharing of 1:

- 7721925095626756828
- 10724818973787827494

Secret sharing of 10865039765974559458:

- 6499945567220489507
- 4365094198754069951





# **Need for interactivity: non-linear computation**

● E.g., **heavy hitters:** Among the measurements uploaded by clients, find the subset that were uploaded at least  $t$  times (for some threshold  $t$ )







facebook.com



### **Data plane**

● Each client shards its measurement into input shares and sends one share to each aggregator





### **Data plane**

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- Aggregators compute aggregate shares, then send their share to the collector





### **Data plane**

- Each client shards its measurement into input shares and sends one share to each aggregator
- Aggregators compute aggregate shares, then send their share to the collector
- Collector unshards the aggregate result





# **Control plane The VDAF: building a box around MPC<br>
Control plane**<br> **Control plane**

● Aggregators interact during aggregation (input validation)





# **Control plane The VDAF: building a box around MPC<br>
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- Aggregators interact during aggregation (input validation)
- Collector might push information to aggregators (heavy hitters with [Poplar](https://eprint.iacr.org/2021/017))





### **Control plane**

- Aggregators interact during aggregation (input validation)
- **•** Collector might push information to aggregators (heavy hitters with **Poplar)**
- Collector might push information to clients (federated learning with [PINE\)](https://arxiv.org/abs/2311.10237)





# **Verifiable Distributed Aggregation Function [\(VDAF\)](https://datatracker.ietf.org/doc/draft-irtf-cfrg-vdaf/)**

Used to compute functions of the form

 $f(\sigma, m_1, ..., m_N) = g(\sigma, m_1) + ... + g(\sigma, m_N)$ 

 $m_1, \ldots, m_N \in$  **Measurements** (chosen by clients)

 ∈ **Aggregation Parameters** (chosen by collector)





# **Privacy**

- Threat model: one aggregator is honest
- Security goal: data minimization
	- Attacker's view of the protocol execution is efficiently simulatable given the aggregate result\*





### **Robustness**

- Threat model: aggregators are honest
- Security goal: collector correctly aggregates honest clients' measurements
	- Aggregate result is efficiently extractable from the attacker's execution (i.e., its random oracle queries)





# **Malicious versus semi-honest security**

- We **must have** privacy against malicious aggregators
- We **don't always need** robustness against malicious aggregators
	- Malicious robustness is nice to have, but not any cost (more parties, more rounds, more bandwidth, etc.)



**Definition 7.2.6** (security in the malicious model): Let  $f$  and  $\Pi$  be as in Definition 7.2.5. Protocol  $\Pi$  is said to securely compute  $f$  (in the malicious model) if for non /.2.5. Protocol 11 is said to securely compare  $f(x) = (A_1, A_2)$  that is admissible<br>every probabilistic polynomial-time pair of algorithms  $\overline{A} = (A_1, A_2)$  that is admissible for the real model (of Definition 7.2.5), there exists a probabilistic polynomial-time pair of algorithms  $\overline{B} = (B_1, B_2)$  that is admissible for the ideal model (of Definition 7.2.4) such that

#### ${\rm \{IDEAL}_{f \overline{R}(z)}(x, y)\}_{x, y, z} \stackrel{c}{\equiv} {\rm \{REAL}_{\Pi} \overline{A}(z)}(x, y)\}_{x, y, z}$

where x, y, z  $\in \{0, 1\}^*$  such that  $|x| = |y|$  and  $|z| = \text{poly}(|x|)$ . (Recall that  $\leq d$ enotes computational indistinguishability by (non-uniform) families of polynomial-size circuits.) When the context is clear, we sometimes refer to  $\Pi$  as a secure implementation of  $f$ .

One important property that Definition 7.2.6 implies is privacy with respect to malicious adversaries. That is, all that an adversary can learn by participating in the protocol, while using an arbitrary (feasible) strategy, can be essentially inferred from the corresponding output alone. Another property that is implied by Definition 7.2.6 is correctness, which means that the output of the honest party must be consistent with an input pair in which the element corresponding to the honest party equals the party's actual input. Furthermore, the element corresponding to the adversary must be chosen<br>eliminated to the hand the second control of the hand to the adversary must be chosen obliviously of the honest party's input. We stress that both properties are easily implied<br>by Definition  $7.26$  kut that the theory of the honest party's input. by Definition 7.2.6, but the latter is not implied by combining the two properties For



- **● VDAF: building a box around MPC**
- **● Building VDAFs**
- Beyond VDAFs
- MPC hot takes  $\bigcirc$



#### **Syntax:**

 $\Pi$ := Prove $(X)$  // proof generation  $V := \text{Query}(X, \Pi; qr)$  // query generation  $d :=$  Decide(V) // decision

**Full linearity:** Query(*X*,  $\Pi$ ;  $q$ r) is equivalent to:

- Split *X*, *II* into shares  $[X]_i$ ,  $[I]_i$  for all *i*
- $[V]_i := \text{Query}([X]_i, [H]_i; qr)$  for all i
- Return  $[V]_1 + ... + [V]_s$



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- $[V]_i := \text{Query}([X]_i, [H]_i; qr)$  for all i
- Return  $[V]_1 + ... + [V]_s$







- PINE: A VDAF for federated learning
	- Aggregating real-valued vectors (**gradients**) with bounded **L2-norm**
	- FLP for L2 norm computation; new techniques for checking correctness of computation
	- More practical than Prio for larger models

**L2 norm:** 
$$
||\mathbf{x}||_2 = ((x_1)^2 + ... + (x_d)^2)^{1/2}
$$



- Constructing FLPs
	- Define validity via a *circuit* C: If  $X \in \mathcal{L}$ , then  $C(X)=0$ ; but if  $X \notin \mathcal{L}$ , then  $C(X) \neq 0$



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```
def counter(x: F) -> F:
    return x * (x-1)# Test
assert counter(0) == 0assert counter(1) == \thetaassert counter(999) != 0
```


- Constructing FLPs
	- Define validity via a (randomized) **circuit** C: If ∈  $\mathcal{L}$ , then C(X)=0; but if  $X \notin \mathcal{L}$ , then C(X)≠0 (w.h.p.)

```
def counter(x: F) -> F:
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assert counter(\theta) == \thetaassert counter(1) == \thetaassert counter(999) != 0
```

```
def histogram(x: list[F], r: list[F]) -> F:
    rng_chk = sum(r[0]**i * x[i] * (x[i]-1) for i in range(len(x)))
    sum chk = sum(x) * (sum(x)-1)
    return r[1] * rng chk + r[1]**2 * sum chk
# Test
assert histogram([0, 0, 0, 0], rand_vec(2)) == 0
assert histogram([0, 0, 1, 0], rand_vec(2)) == 0
assert histogram([0, 0, 999, 0], rand_vec(2)) != 0
assert histogram([1, 0, 1, 0], rand_vec(2)) != 0
```


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def counter(x: F) -> F:
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*Problem:* circuits usually involve non-linear operations ⇒ can't compute these on secret shared data

```
def histogram(x: list[F], r: list[F]) -> F:
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- Constructing FLPs
	- Define validity via a (randomized) **circuit** C: If ∈  $\mathcal{L}$ , then C(X)=0; but if  $X \notin \mathcal{L}$ , then C(X)≠0 (w.h.p.)
	- Proof  $\Pi$  encodes a polynomial  $p$  for which  $p(i)$  is the output of the  $i$ -th non-linear operation

```
def counter(x: F) -> F:
    return p(0)# Test
assert counter(\theta) == \thetaassert counter(1) == \thetaassert counter(999) != 0
```
*Observation:* Polynomial evaluation is linear!

```
def histogram(x: list[F], r: list[F]) -> F:
    rng_chk = sum(r[0]**i * p(i) for i in range(len(x)))
    sum chk = p(\text{len}(x))return r[1] * rng_chk + r[1]*2 * sum_chk# Test
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- Constructing FLPs
	- Define validity via a (randomized) **circuit** C: If ∈  $\mathcal{L}$ , then C(X)=0; but if  $X \notin \mathcal{L}$ , then C(X)≠0 (w.h.p.)
	- Proof  $\Pi$  encodes a polynomial  $p$  for which  $p(i)$  is the output of the  $i$ -th non-linear operation
	- Verifier(s):

```
def counter(x: F) -> F:
    return p(0)# Test
assert counter(\theta) == \thetaassert counter(1) == \thetaassert counter(999) != 0
```
- (Each) Verifier evaluates (its share of)  $C(X)$  using (its share of) p
- Run probabilistic test to check that  $p$  is well-formed (using  $qr$ )



# **Distributed point functions [[GI14](https://www.iacr.org/archive/eurocrypt2014/84410245/84410245.pdf)]**

- Point function:  $f(\alpha) = \beta$  and  $f(X) = 0$  for all  $X \neq \alpha$ 
	- DPF: secret sharing of a point function
		- $(P, K_1, K_2) := \text{Gen}(\alpha, \beta)$
		- $[f(X)]_i = \text{Eval}(P, K_i, X)$  for all  $X_i$ , i





● Prio-style metrics, grouped by **attributes** (user agent, software version, geolocation, etc.) without reducing anonymity set





# **Incremental distributed point functions [[BBCG+21](https://eprint.iacr.org/2021/017)]**

- Incremental point function:  $f(X) = \beta$  for any **prefix** X of  $\alpha \in \{0,1\}^n$  and  $f(X) = 0$  otherwise
	- IDPF: secret-sharing of an incremental point function
		- $(P, K_1, K_2) := \text{Gen}(\alpha, \beta)$
		- $[f(X)]_i = \text{Eval}(P, K_i, X)$  for all  $X_i$ , i





























 $\beta$ 

 $0 \nearrow 1$ 

 $\begin{array}{ccc} 0 & \hspace{1.5cm} & \beta & \hspace{1.5cm} & \end{array}$ 

00  $/$  | 01  $\alpha$ =10 | \ 11

# **Verifiable IDPF [\[MST24](https://eprint.iacr.org/2023/080)]**

- IDPF with verifiability of **one-hotness**
	- $(P, K_1, K_2) := \text{Gen}(\alpha, \beta)$
	- $([f(X_1), ..., f(X_p)]_i, \pi_i) = \text{Eval}(P, K_i, \mathbf{X})$  for all  $\mathbf{X} = (X_1, ..., X_p), i$ 
		- $\pi_1 = \pi_2$  implies  $f(X_1), ..., f(X_p)$  is a one-hot vector
- Also need to verify that the non-zero value is **in-range** 
	- PLASMA [[MST24](https://eprint.iacr.org/2023/080)] solves this for the special case that  $\beta$ =1 (same as Poplar, but with lower round complexity)
	- Mastic  $[MPD+24]$  $[MPD+24]$  $[MPD+24]$  solves the general case via FLP  $\Rightarrow$ weighted heavy hitters, attribute-based metrics

[\[MST24](https://eprint.iacr.org/2023/080)] Mouris et al. "PLASMA: Private, Lightweight Aggregated Statistics against Malicious Adversaries." PETS 2024. [\[MPD+24](https://eprint.iacr.org/2024/221)] Mouris et al. "Mastic: Private Weighted Heavy-Hitters and Attribute-Based Metrics." **In submission.**



# **Boolean-to-arithmetic conversion [[ABJ+22\]](https://eprint.iacr.org/2021/576)**

- Use case: aggregating vectors of counters
- Clients send XOR shares of each counter; aggregators convert to shares in a field suitable for aggregation
	- Much more efficient for clients
	- **Open question:** 2-party conversion that is private in the presence of a malicious aggregator. (Easy in the 3-party, honest-majority setting.)

# **Silently verifiable proofs [[RZCGP24](https://eprint.iacr.org/2024/666)]**

- Extends FLP such that proof verification can be batched across multiple reports
	- Much lower aggregator⇔aggregator bandwidth cost
	- **Open question:** Denial-of-Service (DoS) risk costs increases as the fraction of invalid reports increases
		- Fine for many deployments, but too risky for others
	- Also possible for VIDPF [[MST24](https://eprint.iacr.org/2023/080)]





- **● VDAF: building a box around MPC**
- **● Security goals for VDAFs**
- **● Building VDAFs**
- **● Beyond VDAFs**
- MPC hot takes  $\bigcirc$



# **Heavy hitters via threshold secret sharing [[DSG+22](https://arxiv.org/abs/2109.10074)]**

- Basic idea:
	- Each client generates a  $t$ -of- $n$  secret share of a key to encrypt its measurement
	- $\bullet$  After receiving *t* shares, aggregator can recover the key and decrypt
- Based on an Oblivious PRF (OPRF) [[RFC 9497](https://datatracker.ietf.org/doc/rfc9497/)]
- With more recent techniques  $[INT24]$ , achieves the same level of privacy as Poplar (in a slightly different threat model)
- Achieving robustness is expensive





[\[DSG+22\]](https://arxiv.org/abs/2109.10074) Davidson et al. "STAR: Secret Sharing for Private Threshold Aggregation Reporting." CCS 2022. [\[RFC 9497](https://datatracker.ietf.org/doc/rfc9497/)] Davidson et al. "Oblivious Pseudorandom Functions (OPRFs) Using Prime-Order Groups." [\[LNT24](https://eprint.iacr.org/2024/320)] Li et al. "POPSTAR: Lightweight Threshold Reporting with Reduced Leakage." **In submission.**



### **Sparse histograms**

- Clients hold pairs  $(\alpha, \beta)$ : for each index  $\alpha$  held by at least one client, compute the sum of the values  $\beta$
- Protocol of  $[BGR+24]$ 
	- **Based on OPRF and multiplicative** homomorphic encryption
	- Differentially privacy baked in by default ⇒ much better utility than Poplar with differential privacy

**Differential privacy**: aggregate should not depend "too much" on any one measurement



*Credit: [Paille](https://www.flickr.com/photos/paille-fr/24559019804) // CC BY-SA 2.0.*



# **Joining data sources**

- Last-touch attribution: count the number of purchases attributable to an ad
	- Put purchases and ad impressions in a database: for each purchase, find the most recent ad impression
	- [IPA](https://github.com/patcg-individual-drafts/ipa/blob/main/IPA-End-to-End.md) ("Interoperable Private Attribution"): Sorting via the 3-party, honest majority computation [[CHI+19](https://eprint.iacr.org/2019/695)]







- **● VDAF: building a box around MPC**
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## **MPC is crypto + distributed systems**

- While some requirements like memory, CPU, and bandwidth are well-documented in the literature, many other requirements are not well understood
	- Strong versus eventual consistency
	- Load balancing across machines
	- Moving workloads between machines





### **Number of parties**

- 2 parties fits neatly into client-server architecture of HTTP
- 3 parties is more complex, but workable
- ≥4 parties is probably too much coordination







### **Number of rounds**

- 1 round: Complete aggregation in a single HTTP request
- $\bullet$   $\geq$  2 rounds: Have to keep state across HTTP requests; less flexibility in the server architecture





# **Differential privacy should be baked in**

- Especially important when the protocol has leakage (e.g., heavy hitters)
- Generic composition of VDAF with some DP mechanism usually has sub-optimal utility
- **Challenge:** Securely sample shares of discrete Gaussian or Laplace with low communication cost, in the 2-party setting [[KKL+23\]](https://petsymposium.org/popets/2023/popets-2023-0086.pdf)

THEOREM 1 ( $\Pi$  is  $\epsilon$ -SIM-CDP). Let S be the simulator defined in FIGURE 1. Then:

(ii) For every  $\Pi$ -attacker A there exists a Foo-attacker B such that  $\mathbf{Adv}_{\Pi}^{\text{sim-cdp}}(A, S) \leq \mathbf{Adv}_{\text{Eoo}}^{\text{bar}}(B).$ 

**Differential privacy**: aggregate should not depend "too much" on any one measurement



*Credit: [Paille](https://www.flickr.com/photos/paille-fr/24559019804) // CC BY-SA 2.0.*

<sup>(</sup>i) S is  $\epsilon$ -DP.



### **Thanks!**

- Join the mailing list:<https://www.ietf.org/mailman/listinfo/Ppm>
- Join #ppm in the IETF slack:<https://ietf.slack.com/>
- Base drafts:
	- DAP: <https://datatracker.ietf.org/doc/draft-ietf-ppm-dap/>
	- VDAF:<https://datatracker.ietf.org/doc/draft-irtf-cfrg-vdaf/>
- Individual drafts in progress for new VDAFs, differential privacy, dealing with Sybil attacks, and more!



# **Constructing IDPFs**

•  $P$ ,  $K_1$  and  $P$ ,  $K_2$  are concise representations of binary trees:  $\alpha$ -path nodes are **secret shares** of  $\beta$ ; and off-path nodes are **equal** 





# **Function secret sharing**

- FSS [[BGI16](https://eprint.iacr.org/2018/707)]: split *f* into shares such that  $[f(X)]_1$ , ...,  $[f(X)]_s$  can be evaluated for any *X* 
	- Possible to construct efficient schemes for specific classes of functions
	- Transforming privacy-only FSS to verifiable FSS
		- Arithmetic sketching [[BBCG+23\]](https://eprint.iacr.org/2023/1012) generalizes sketching scheme from Poplar for achieving robustness with IDPFs



### **Standardized DP mechanisms**

- Bridging the DP and MPC communities:
	- Secret-sharing the noise [[EIKN22](https://eprint.iacr.org/2022/1391), [KKL+23](https://petsymposium.org/popets/2023/popets-2023-0086.php)]
	- Algorithms for sampling from non-uniform distributions (e.g., discrete Gaussian [[CKS20](https://arxiv.org/abs/2004.00010)])
	- Collective experience with privacy/utility trade-off

[\[EIKN21\]](https://eprint.iacr.org/2022/1391) Eriguchi et al. "Efficient Noise Generation Protocols for Differentially Private Multiparty Computation." FC 2021. [\[KKL+23\]](https://petsymposium.org/popets/2023/popets-2023-0086.php) Keeler et al. "DPrio: Efficient Differential Privacy with High Utility for Prio." PETS 2023. [\[CKS20\]](https://arxiv.org/abs/2004.00010) Canonne et al. "The Discrete Gaussian for Differential Privacy." NuerIPS 2020.